Final Report (Project ID # 23270225): Experimental Observation of Stochastic Resonance in Magnetically Driven Damped Duffing Oscillator

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(Dated: December 7, 2021)

Abstract

We present results of numerical and experimental studies of Stochastic Resonance (SR) in the mechanical model of the magnetically driven Duffing oscillator. In this project, we developed numerical model of the Duffing oscillator and designed computer controlled magneto-mechanical oscillator, which is described by the Duffing equation. The developed system was used to experimentally observed stochastic resonance.

I. PROJECT MAIN OBJECTIVES AND RESEARCH METHODS

The **main goals** of the project were 1) to build a Damped Driven Duffing Oscillator setup with the external stochastic force and 2) to investigate its behavior, including experimental observation of stochastic resonance. To achieve these goals the following tasks were identified:

1. Development of numerical simulation of SR phenomenon in bistable system described by the Duffing equation
2. Design the experimental setup to observe SR phenomenon
3. Design the LabVIEW controlled DAQ to collect experimental data
4. Collect and analyze experimental data

II. PROJECT RESULTS

A. Task 1: Numerical Simulation of the Duffing Oscillator with External Stochastic Force

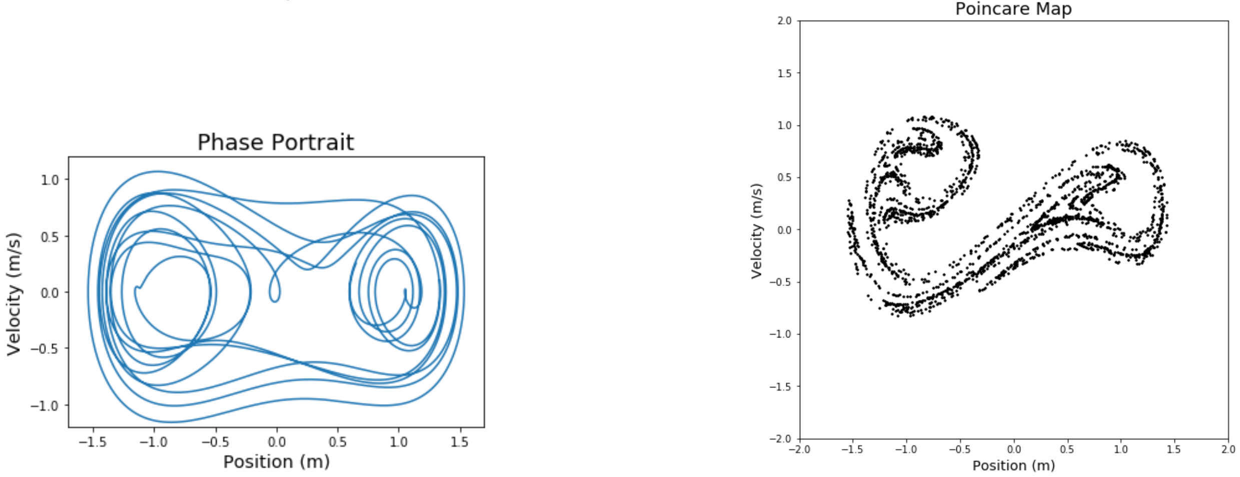
Motion of the damped driven Duffing oscillator with an external and stochastic forces is described by

$$\ddot{x} + \delta\dot{x} + ax + bx^3 = F(t) + \xi(t), \tag{1}$$

where δ is a damping coefficient, $F(t)$ is an external driving force, and $\xi(t)$ is a stochastic force.

To understand experimental results we first investigated the developed setup numerically using Eq. (1) with experimentally measured parameters a and b. We obtained phase portraits, Poincaré maps, when the external driving force is harmonic $F(t) = A\cos(\omega t)$ and no noise is present. Computational studies were performed with Python ran on a computing cluster using MPI as well as the *Jupyter Notebook* environment.

To solve numerically the ODE (1) without noise, the Runge–Kutta methods (RK4) provided by the package *Scipy* was used. Example of the obtained phase portrait and Poincare map are shown in Fig. 1.



(a) Example of phase portrait

(b) Example of Poincare map

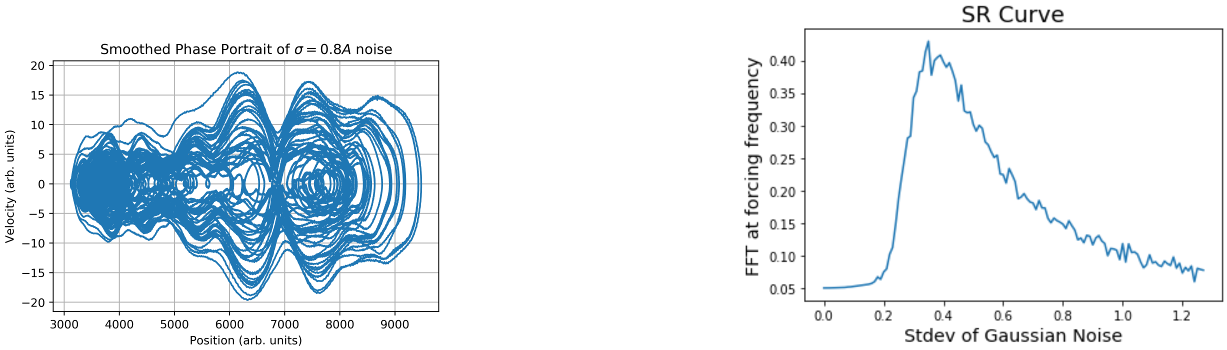
FIG. 1: Results of the numerical analysis of the Duffing oscillator without presence of stochastic force.

The RK4 method does not work for simulating stochastic differential equations such as the Duffing equation with added external stochastic force (or noise), $\xi(t)$ in (1). This is due to the fact that the RK4 methods uses continuity assumption of the ODE, which is clearly violated by the noise. Instead, a stochastic Runge-Kutta method implemented in the Python package *sdeint* was used to simulate the stochastic differential equation (1) when noise is present. Phase portrait of the Duffing oscillator with Gaussian noise is shown in Fig. 2a. Numerical simulation confirmed that the Duffing oscillator is an excellent system to observe stochastic resonance in a presence of an external noise. Calculated SR curve is shown in Fig. 2b.

B. Task 2: Design and Development of the Experimental Setup to Observe SR

The proposed experimental setup is based on the one presented in Guillermo Donoso and Celso L Ladera 2012 *Eur. J. Phys.*, **33**, 1473. The model consists of a disk magnet ($r = 0.5$ in, $h = 0.25$ in) with dipole moment $\mu = 5.26$ Am^2 attached to a spring that oscillates vertically in the magnetic field generated by a small coil (see Fig. Fig. 5a). The small coil dimensions are $r_A = 2$ cm, $L_A = 3.8$ cm, $N_A = 20$.

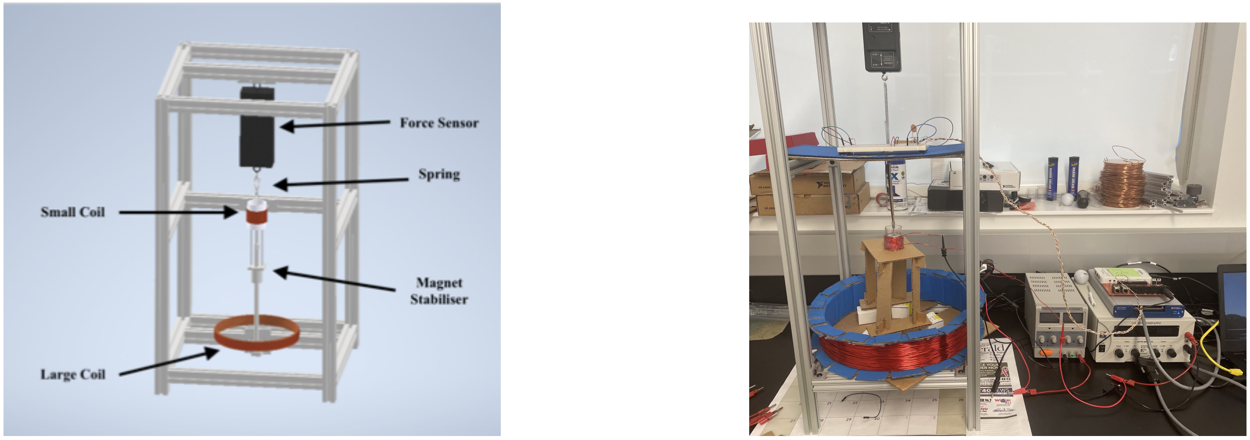
The spring constant and damping parameter of the system were determined by collecting



(a) Phase portrait with Gaussian noise.

(b) Simulated stochastic resonance curve

FIG. 2: Results of the numerical analysis of the Duffing oscillator with presence of noise.



(a) CAD drawing

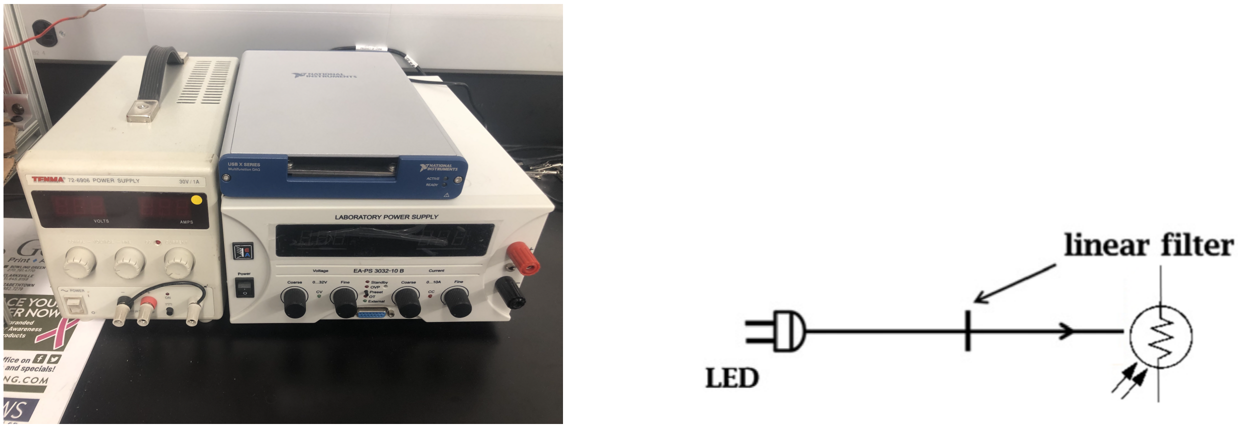
(b) Assembled experimental setup

FIG. 3: Duffing oscillator designed and developed in this study

oscillation data with the magnetic field generated by the small coil. In this case, the system exhibited a damped harmonic oscillations. The parameters of the Duffing equation, δ, a and b from Eq. (1), were determined by fitting obtained data.

Combining magnetic forces from the small coil and restoring elastic spring force, the total force acting on magnet can be calculated analytically.

The external periodic and stochastic forces are generated by a second coil (large coil in Fig.4a), $r_B = 17.25$ cm, $L_B = 7.3$ cm, $N_B = 270$. The coil is located below the system such that the magnet is located at the inflection point of the magnetic field. The `EA-PS 3032-10 B` power supply by `Elektro-Automatik` (shown in Fig. 4a) capable of supplying 32V and up to 10A was used to generate magnetic force from the large coil. In order to apply the random noise force, a rapidly varying current is forced through the coil. Because the coil has a self-inductance, there is a limit on how fast the current can be varied. We estimated that the large coil has an inductance of $0.1H$ and resistance of 3.3Ω. Hence, the noise can be introduced at a frequency of about 15 Hz. As long as the forcing frequency applied to the system is much slower than the frequency of applied noise, the integrity of the noise is preserved and will cause stochastic resonance. The position of the magnet was measured using a phototransistor (BPV11)-LED pair. The LED transmits white light through a linear optical filter printed on an acetate sheet attached to the magnet suspension system. The collector of the phototransistor is connected to a +5V input and the current out of the emitter is close to linearly proportional to the light incident on the phototransistor. This current is sent through a low pass filter to ground and the voltage difference across the resistor in the filter is measured.



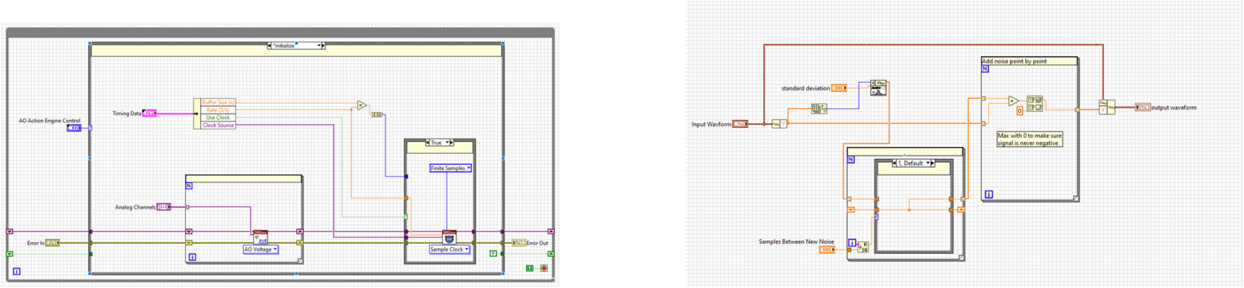
(a) EA-PS 3032-10 B power supply and NI USB-6341 DAQ

(b) BPV11-LED pair

FIG. 4: Power supply and phototransistor-LED pair for position measurement

C. Task 3: Design, build and programming of the LabVIEW based DAQ

A LabVIEW data acquisition code was created using `USB-6341` to acquire position data from phototransistor as well to control `EA-PS 3032-10 B` power supply unit which provides current through the large coil. The code uses the usual paradigm of functional global variables and queued state machines to provide the user a simple interface for interacting with the system. The full LabVIEW project is available at the Project code depository. Example of LabVIEW interface/code is shown in the figure below:



Max with 0 to make sure signal is never negative

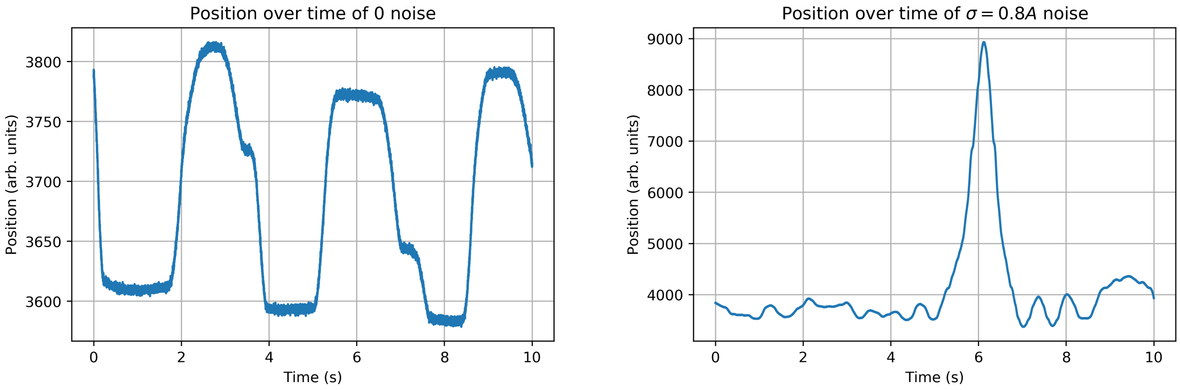
(a) LabVIEW DAQ initialization control

(b) Gaussian noise generation

FIG. 5: LabVIEW based DAQ control code

D. Task 4: Experimental Results and Data Analysis

Using developed experimental setup, the position of the disk magnet was measured with different stochastic force magnitudes. Stochastic force was characterized by standard deviation (σ) of noise magnetic field generating current in the large coil. Example of the obtained data is shown in Fig. 6.



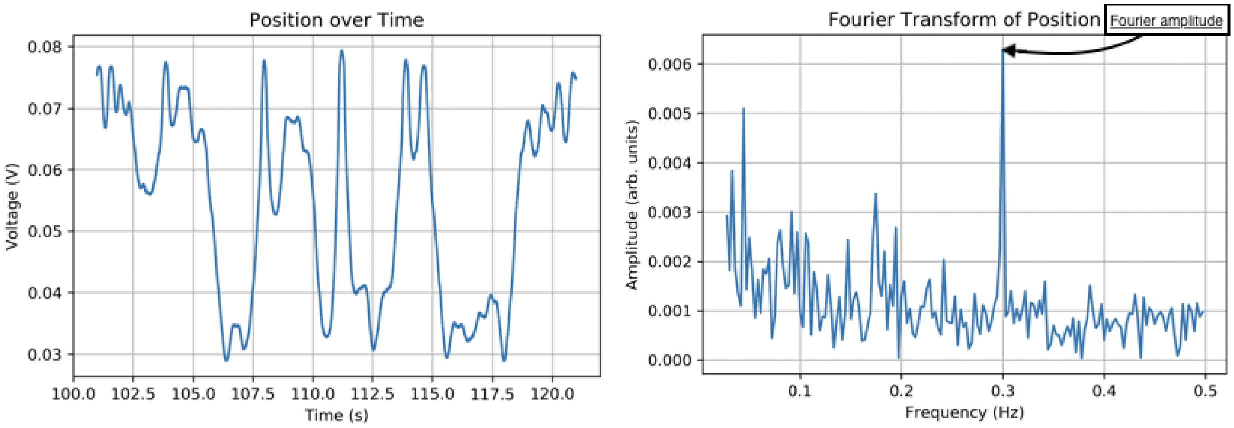
(a) Magnet position without noise

(b) Magnet position with Gaussian noise

FIG. 6: Measured with phototransistor-LED pair magnet position

The general data analysis and visualization routine can be described as follows:

1. Collect experimental data on the magnet position for various levels of Gaussian noise. The data is collected as a time-series.



(a) Measure position with noise at $\sigma = 0.4$ A

(b) Fourier spectrum

FIG. 7: Magnet position as a function of time and corresponding Fourier spectrum. External harmonic force $F(t) = A\cos(\omega t)$ was applied, where $\omega = 0.3$ Hz.

2. Perform Fourier Transformation on the collected data to obtained amplitude in frequency domain of the time-series signal which corresponds to transitions between magnet equilibria positions at the forcing force frequency.

3. Plot value of the Fourier amplitude as a function Gaussian noise characterized by the standard deviation

The procedure was repeated for various magnitudes of stochastic force. As a result, Fourier amplitude for each noise level was obtained (see Fig. 8).

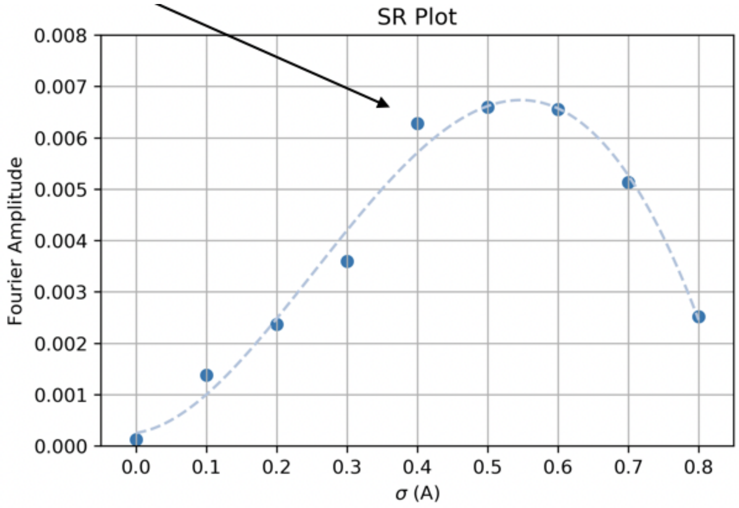


FIG. 8: Experimentally obtained stochastic resonance curve. Arrow shows Fourier amplitude data point extracted from the data shown in Fig. 7

III. SUMMARY OF THE DISBURSEMENTS

The obtained funds were used to purchase various materials and supplies used for assembling the magneto-mechanical Duffing oscillator (aluminum framing, magnetic wires, disk magnets, springs), as well as to cover salary of one undergraduate student (Lars Hebenstiel).

IV. LIST OF PRESENTATIONS

1. Lars Hebenstiel and Ivan Novikov, ”Experimentally Modeling Stochastic Resonance in the Duffing Oscillator”, Posters-at-the-Capitol, 2020

   Abstract Stochastic Resonance (SR) is a phenomenon that has been observed in various disciplines including climate science, mechanical and electrical engineering, optics, and various chemical and sociological models. Because it has been shown that energy harvesting is possible using a nonlinear system, it was proposed that SR can be used to increase the amount of collected energy. SR occurs when the optimal amount of white noise is added to a bistable or nonlinear system such that the signal to noise ratio is maximized. Energy harvesters rely mainly on ambient energy sources such as temperature fluctuations or electromagnetic waves as their source of power, so it is possible random fluctuations in these can be controlled to cause SR. The Damped Driven Duffing Oscillator (DDDO) is a bistable system with a damping force, which exhibits chaotic behavior when a periodic external force is applied. To observe SR in the DDDO an external random force or white noise is introduced into the system. The goals of this project are to build a DDDO system experimental setup controlled by LabVIEW modules and to experimentally study SR. As the first step, numerical methods have been used to calculate analytical solutions of the DDDO. These solutions were visualized as phase portraits and Poincaré maps were plotted. The phase portrait shows both the position and velocity in an intuitive way; the Poincaré map of the DDDO shows the periodic recurrence of the oscillator, a result that cannot be observed in the phase portrait due to the chaotic nature of the oscillator. Data analysis and visualization was done in the Jupyter Notebook environment, which allows our work to be easily shared and worked with by other researchers. The project is supported by The Gatton Academy at Western Kentucky University.

2. Doug Harper, Lars Hebenstiel, and Ivan Novikov, "Progress and Development of a Mechanical Damped Driven Duffing Oscillator", 2020 Virtual KAS Annual Meeting, November 6-7, 2020

   Abstract The Duffing Oscillator (DO) is a bistable, nonlinear oscillator initially described by Georg Duffing in Erzwungene schwingungen bei veränderlicher eigenfrequenz und ihre technische bedeutung. Braunschweig, F. Vieweg & sohn (1918). Because of its nonlinear nature, the DO exhibits stochastic resonance, which has been observed in ring lasers, electron paramagnetic resonance and various other models. In this talk, we present the progress and development of a magnetically driven mechanical model of the DO. The design is inspired by the one proposed in Donoso, Ladera, Eur. J. Phys. 33 (2012). The design features two coils and a spring with a high- power rare earth magnet attached oriented vertically to generate the quartic Duffing potential and a sinusoidal forcing function. To increase accuracy, we introduced a number of improvements to constrain the horizontal oscillations of the magnet. To control the experimental apparatus, a new LabVIEW based DAQ system was developed.

3. Doug Harper, Lars Hebenstiel and Ivan Novikov, "Experimental and Numerical Studies of the Mechanical Damped Driven Duffing Oscillator", 87th annual meeting of the Southeastern Section of the APS, Volume 65, Number 19, November 5–6, 2020 (virtual)

   Abstract The Duffing Oscillator (DO) is a bistable, nonlinear oscillator derived from a double well potential which exhibits various phenomena such resonance and stochastic resonance (SR). SR occurs when a system experiences resonance due to some amount of noise being added to the system, with some optimal amount of noise for the most resonance. These phenomena have been observed in ring lasers, electron paramagnetic resonance and various other models. In this talk, we present the experimental results obtained with a magnetically driven mechanical model of the DO recently proposed. The system consists of high-power magnet attached to a spring and placed inside a solenoid. The potential energy of this system is described through use of binomial expansion as a fourth order potential. The current through the second coil placed underneath the experimental setup is varied to provide external periodic and white noise forces. The oscillator's position is measured with a force probe, and with a grayscale transparency, photo-resistor/LED system which measures the intensity of light passing through the semitransparent sheet. LabVIEW control systems are used to vary the current through the coils and acquire data from the position sensors.

4. Doug Harper, Lars Hebenstiel and Ivan Novikov, "Setup of a Magnetically Driven Duffing Oscillator to Measure Stochastic Resonance in Undergraduate Physics Labs", 88th Annual Meeting of the Southeastern Section of the APS, November 18–20, 2021 (Tallahassee, FL)

   Abstract The Duffing Oscillator (DO) is a bistable, nonlinear oscillator initially described by Georg Duffing in 1918. Due to its nonlinear nature, the DO is an excellent system to study stochastic resonance, a phenomenon also occurring in ring lasers, electron paramagnetic resonance and other non-linear systems. Stochastic resonance is a phenomenon whereby a system experiences resonance due to noise being added to it. Optimal stochastic resonance occurs when the optimal amount of noise is added. In this talk, we present the progress in development of a magnetically driven mechanical model of the DO. The design is inspired by the one proposed in *Donoso, Ladera, Eur. J. Phys. 33 (2012).* The setup is powered by a LabVIEW DAQ system which can be easily controlled by any modern computer. With it, students can experimentally observe phase portraits, poincare maps, bifurcation diagrams, and stochastic resonance. This allows students to familiarize themselves with topics such as chaos, bifurcation and stochastic resonance.

5. Doug Harper, Lars Hebenstiel and Ivan Novikov, "Experimental Observation of Stochastic Resonance in a Magnetically Driven Mechanical Duffing Oscillator", 2021 Virtual KAS Annual Meeting, November 5-6, 2021. Presentation received 2nd place in Student Competition in Physics and Astronomy Section.

   Abstract The Duffing Oscillator (DO) is a bistable, nonlinear oscillator initially described by Georg Duffing in 1918. Due to its nonlinear nature, the DO is an excellent system to study a stochastic resonance, a phenomenon also occurring in ring lasers, electron paramagnetic resonance and other non-linear systems. Stochastic resonance (SR) is a phenomenon whereby a system experiences resonance due to noise being added to it. Optimal stochastic resonance occurs when the optimal amount of noise is added. In this talk, we present the progress in development of a magnetically driven mechanical model of the DO. The design is inspired by the one proposed in Donoso, Ladera, Eur. J. Phys. 33 (2012), where a high-power rare earth magnet is attached to a spring and placed inside a small copper coil. Potential energy of the system can be described by a fourth order function and is called Duffing potential. The second coil placed underneath of the experimental setup provides an external periodic and white noise forces. The external force magnitude is controlled via the electrical current passing through this coil. Data acquisition and system control is powered by a LabVIEW DAQ system. We show here our experimental setup and procedure as well as the categorization of SR

6. Lars Hebenstiel, ”Experimental Observation of Stochastic Resonance in a Magnetically Driven Mechanical Duffing Oscillator”, WKU Physics and Astronomy Colloquium, Bowling Green, KY; November 28, 2021.

   Abstract The Duffing Oscillator (DO) is a bistable, nonlinear oscillator initially described by Georg Duffing in 1918. Due to its nonlinear nature, the DO is an excellent system to study stochastic resonance, a phenomenon also occurring in ring lasers, electron paramagnetic resonance and other non-linear systems. Stochastic resonance (SR) is a phenomenon whereby a system experiences resonance due to noise being added to it. Optimal stochastic resonance occurs when the optimal amount of noise is added. In this talk, we present the progress in development of a magnetically driven mechanical model of the DO. The design is inspired by the one proposed in Donoso, Ladera, Eur. J. Phys. 33 (2012), where a high-power rare earth magnet is attached to a spring and placed inside a small copper coil. Potential energy of the system can be described by a fourth order function and is called Duffing potential. The second coil placed underneath of the experimental setup provides an external periodic and white noise forces. The external force magnitude is controlled via the electrical current passing through this coil. Data acquisition and system control is powered by a LabVIEW DAQ system. We show here our experimental setup and procedure as well as the categorization of SR.

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